Resistive and magnetized accretion flows with convection

Kazem Faghei and Mobina Omidvand

Abstract We considered the effects of convection on the radiatively inefficient accretion flows (RIAF) in the presence of resistivity and toroidal magnetic field. We discussed the effects of convection on transports of angular momentum and energy. We established two cases for the resistive and magnetized RIAFs with convection: assuming the convection parameter as a free parameter and using mixing-length theory to calculate convection parameter. A self-similar method was used to solve the integrated equations that govern the behavior of the presented model. The solutions showed that the accretion and rotational velocities decrease by adding the convection parameter, while the sound speed increases. Moreover, by using mixing-length theory to calculate convection parameter, we found that the convection can be important in RIAFs with magnetic field and resistivity.

Keywords accretion, accretion discs, convection, magnetohydrodynamics: MHD

1 Introduction

The existence of radiatively inefficient accretion flows (RIAFs) have been confirmed in low-luminosity state of X-ray binaries and nuclei of galaxies (Narayan et al. 1996; Esin et al. 1997; Di Matteo et al. 2003; Yuan et al. 2003). It was understood that RIAFs are likely to be convectively unstable in the radial direction due to the inward increase of the entropy of accreting gas (Narayan & Yi 1994). Moreover, hydrodynamical and magetohydrodynamical simulations of low-viscosity RIAFs have confirmed these flows are convectively unstable (e. g. Igumenshchev et al. 1996; Stone et al. 1999; Machida et al. 2001; Hawley & Balbus 2002; McKinney & Gammie 2002; Igumenshchev et al. 2003). Self-similar or global solutions for convection-dominated accretion flows (CDAFs) were presented by several authors (e. g. Narayan et al. 2000; Quataert & Gruzinov 2000; Abramowicz et al. 2002; Lu et al. 2004; Zhang & Dai 2008).

Igumenshchev et al. (2003) studied the resistive MHD simulations of RIAFs onto black holes. They assumed two cases for the geometry of the injected magnetic field: pure toroidal field and pure poloidal field. They found that in the case of pure toroidal magnetic field, the accreting gas forms a nearly axisymmetric, geometrically thick, turbulent accretion disc. Moreover, their solutions represented that the flow resembles in many respects CDAFs found in previous numerical and analytical investigations of viscous hydrodynamic flows. Zhang & Dai (2008) investigated the effect of magnetic field on RIAFs with convection by a semi-analytically method. By exploit of α-prescription for viscosity and convection, they used two methods to study of magnetized flows with convection, i.e. they take the convective coefficient α_c as a free parameter to discuss the effects of convection for simplicity. They also established the α_c-α relation for magnetized flows using the mixing-length theory and compare this relation with the non-magnetized case. They found that the magnetic field makes the α_c-α relation be distinct from that of non-magnetized flows.

Since the importance of toroidal magnetic field and resistivity in accretion flows have been confirmed observationally (see Faghei 2011 and references therein), Faghei (2011) considered the steady, radially self-similar solutions of accretion flows in the presence of the toroidal magnetic field and the resistivity. However, he ignored the effects of convection in his model. Generally semi-analytical studies of magnetized CDAFs are related to non-resistive magnetized CDAFs (e. g.

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Zhang & Dai 2008) and the resistive and magnetized CDAF was studied in MHD simulations (e. g. Igumenshchev & Narayan 2002; Hawley & Balbus 2002; Igumenshchev et al. 2003). Thus, it will be interesting to study the effects of resistivity on RIAFs with convection. Here, we adopt the presented solutions by Narayan et al. (2000) and Faghei (2011). Similar to Narayan et al. (2000), we will discuss the effects of convection on angular momentum and energy equations. The paper is organized as follow. In section 2, the basic equations of constructing a model for quasi-spherical magnetized RIAFs with convection will be defined. In section 3, a self-similar method for solving equations which govern the behavior of the accreting gas was utilized. The summary of the model will appear in section 4.

2 Basic Equations

Analytical theory of CDAF is based on a self-similar solution of a simplified set of equations describing RIAFs. We adopted the presented solutions by Narayan et al. (2000) and Faghei (2011). By using spherical coordinate (r, θ, φ) centered on a accreting object, let us consider stationary, axisymmetric, quasi-spherical equations describing an accretion flow onto the black hole of mass M. For the sake of simplicity, the general-relativistic effect has been neglected and the gravitational force on a fluid is characterized by Newtonian potential of a point mass, $\psi = -GM/r$. As magnetic fields, we consider only toroidal fields, B_φ.

Under these assumptions, the continuity equation with mass loss is

$$\frac{1}{r^2}\frac{d}{dr}(r^2 \rho v_r) = \dot{\rho}, \tag{1}$$

where ρ, v_r and $\dot{\rho}$ are the density, the accretion velocity ($v_r < 0$), and the mass-loss per unit volume, respectively.

The radial momentum equation is

$$v_r \frac{dv_r}{dr} = r\left(\Omega^2 - \Omega_K^2\right) - \frac{1}{\rho}\frac{d}{dr}(\rho c_s^2) - \frac{c_A^2}{r} - \frac{1}{2\rho}\frac{d}{dr}(\rho c_A^2), \tag{2}$$

where c_s is sound speed, which is defined as $c_s^2 \equiv p_{gas}/\rho$, with being p_{gas} as the gas pressure, Ω is the angular velocity, $\Omega_K [= \left(GM/r^3\right)^{1/2}]$ is the Keplerian angular velocity, and c_A is the alfven speed, which is defined as $c_A^2 \equiv B_\varphi^2/4\pi\rho = 2p_{mag}/\rho$, with being p_{mag} as the magnetic pressure. The ram-pressure term $v_r dv_r/dr$ and last two terms due to the magnetic field in this equation were ignored in the self-similar CDAF model of Narayan et al. (2000), while we include them here in order to consider their effects.

The angular momentum equations can be written in the form of the balance of advection and diffusion transport terms (Narayan et al. 2000),

$$\rho v_r \frac{d}{dr}(r^2\Omega) = \frac{1}{r^2}\frac{d}{dr}\left[\nu \rho r^4 \frac{d\Omega}{\partial r}\right] + \frac{1}{r^2}\frac{d}{dr}\left[\nu_c \rho r^{(5+3g)/2}\frac{d}{dr}\left(\Omega r^{3(1-g)/2}\right)\right], \tag{3}$$

where the two terms of right hand side represent the angular momentum transport by viscosity and convection. Here, ν is the kinematic viscosity coefficient, ν_c is the convective diffusion coefficient, and g is the parameter to determine the condition of convective angular momentum transport. When $g = 1$, the flux of angular momentum due to convection is

$$\dot{J}_c = -\nu_c \, \rho \, r^4 \, \frac{d\Omega}{dr}. \tag{4}$$

The above equation implies that the convective angular momentum flux is oriented down the angular velocity gradient. For a quasi-Keplerian angular velocity, $\Omega \propto r^{-3/2}$, angular momentum is transported outward. When $g = -1/3$, the convective angular momentum flux can be written as

$$\dot{J}_c = -\nu_c \, \rho \, r^2 \, \frac{d\left(\Omega r^2\right)}{dr}. \tag{5}$$

This equation represents that the convective angular momentum flux is oriented down the specific angular momentum gradient. For a quasi-Keplerian angular velocity, $\Omega \propto r^{-3/2}$, angular momentum is transported inward. Generally, convection transports angular momentum inward (or outward) for $g < 0$ (or > 0), and the specific case $g = 0$ corresponds to zero angular momentum transport (Narayan et al. 2000).

In this paper, we assume the kinematic coefficient of viscosity and the magnetic diffusivity due to turbulence in the accretion flow. So, we use these parameters in analogy to the α-prescription of Shakura & Sunyaev (1973) for the turbulent,

$$\nu = P_m \eta = \alpha \frac{c_s^2}{\Omega_K}, \tag{6}$$

where P_m is the magnetic Prandtl number of the turbulence, which assumed to be a constant less than unity, η is the magnetic diffusivity, and α is a free parameter less than unity. For the convective diffusion coefficient, ν_c, we adopt the assumptions of Narayan et al. (2000) and Lu et al. (2004) that all transport phenomena due

to convection have the same diffusion coefficient, which is defined as

$$\nu_c = \left(\frac{L_M^2}{4}\right)\sqrt{-N_{eff}^2}, \tag{7}$$

where L_M is the characteristic mixing length and N_{eff} is the effective frequency of convective blobs. The characteristic mixing length L_M in terms of the pressure scale height, H_p, can be written as

$$L_M = 2^{-1/4} l_M H_p, \qquad H_p = -\frac{dr}{d\ln p_{gas}}, \tag{8}$$

where l_M is the dimensionless mixing-length parameter and its amount is estimated to be equal to $\sqrt{2}$ in ADAFs (Narayan et al. 2000; Lu et al. 2004). the effective frequency of convective blobs, N_{eff}, is given by

$$N_{eff}^2 = N^2 + \kappa^2, \tag{9}$$

where N is Brunt-Väisälä frequency, which is defined as

$$N^2 = -\frac{1}{\rho}\frac{dp_{gas}}{dr}\frac{d}{dr}\ln\left(\frac{p_{gas}^{1/\gamma}}{\rho}\right), \tag{10}$$

and κ is epicyclic frequency, which is defined as

$$\kappa^2 = 2\Omega^2\frac{d\ln(\Omega r^2)}{d\ln r}. \tag{11}$$

For a non-Keplerian flows $\kappa \neq \Omega$, while for a quasi-Keplerian ($\Omega \propto r^{-3/2}$), $\kappa = \Omega$ (Narayan et al. 2000; Lu et al. 2004). Convection appears in flows with $N_{eff}^2 < 0$. We also write the convective diffusion coefficient in the form similar to usual viscosity of Shakura & Sunyaev (1973),

$$\nu_c = \alpha_c\frac{c_s^2}{\Omega_K} \tag{12}$$

where α_c is a dimensionless coefficient that describes the strength of convective diffusion. The α_c coefficient can be obtained by equations (8) and (13)

$$\alpha_c = \frac{\Omega_K}{c_s^2}\left(\frac{L_M^2}{4}\right)\sqrt{-N_{eff}^2}. \tag{13}$$

The energy equation is

$$\rho v_r T\frac{ds}{dr} \equiv \rho v_r\left[\frac{1}{\gamma-1}\frac{dc_s^2}{dr} - \frac{c_s^2}{\rho}\frac{d\rho}{dr}\right] = Q_{diss} + Q_{conv} - Q_{rad}, \tag{14}$$

where T is the temperature, s is the specific entropy, γ is the ratio of specific heats, Q_{diss} is dissipative heating rate, Q_{rad} is the radiative cooling rate, and $Q_{conv} = -\nabla\cdot\mathbf{F}_{conv}$, with being $F_{conv}[= -\rho\nu_c T ds/dr]$ as the outward energy flux due to convection. For the right hand side of the energy equation, we can write

$$Q_{adv} = fQ_{diss} - \frac{1}{r^2}\frac{d}{dr}\left(r^2 F_{conv}\right), \tag{15}$$

where Q_{adv} is the advective transport of energy, and $f[= 1 - Q_{rad}/Q_{diss}]$ is the advection parameter. The parameter f measures the degree to which the flow is advection-dominated (Narayan & Yi 1994). The dissipative heating rate can be written as

$$Q_{diss} = (\nu + g\nu_c)\rho r^2\left(\frac{\partial\Omega}{\partial r}\right)^2 + \frac{\eta}{4\pi}\mathbf{J}^2, \tag{16}$$

where the right-hand side terms are heating rate due to viscosity, convection, and resistivity, respectively. In above equation, $\mathbf{J}[= \nabla\times\mathbf{B}]$ is the current density, with being $\mathbf{B}$ as the magnetic field.

Finally, the induction equation with creation/escape of magnetic field can be written as

$$\frac{1}{r}\frac{d}{dr}\left[r v_r B_\varphi - \eta\frac{d}{dr}(rB_\varphi)\right] = \dot{B}_\varphi. \tag{17}$$

where B_φ is the toroidal component of magnetic field and $\dot{B}_\varphi$ is the field escaping/creating rate due to a magnetic instability or dynamo effect. This induction equation is rewritten as

$$\dot{B}_\varphi = \frac{1}{r}\frac{d}{dr}\left[\sqrt{4\pi\rho c_A^2}\left(rv_r - \frac{\alpha}{4\chi P_m}\frac{1}{r\rho\Omega_K}\frac{d}{dr}(r^2\rho c_A^2)\right)\right], \tag{18}$$

where χ is the ratio of the magnetic pressure to the gas pressure, which is defined by

$$\chi = \frac{p_{mag}}{p_{gas}} = \frac{1}{2}\left(\frac{c_A}{c_s}\right)^2. \tag{19}$$

3 Self-Similar Solutions

We seek self-similar solutions in the following form (e.g. Narayan & Yi 1994; Akizuki & Fukue 2006)

$$v_r(r) = -c_1\alpha\sqrt{\frac{GM_*}{r}} \tag{20}$$

$$\Omega(r) = c_2\sqrt{\frac{GM_*}{r^3}} \tag{21}$$

$$c_s^2(r) = c_3\frac{GM_*}{r} \tag{22}$$

$$c_A^2(r) = \frac{B_\varphi^2}{4\pi\rho} = 2\chi c_3 \frac{GM_*}{r} \tag{23}$$

where c_1, c_2, and c_3 are dimensionless constant to be determined. We use a power-law relation for density

$$\rho(r) = \rho_0 r^\lambda, \tag{24}$$

where ρ_0 and λ are constant. Using equations (20)-(24), the mass-loss rate and the magnetic field escaping/creating rate can be written as

$$\dot\rho(r) = \dot\rho_0 r^{\lambda-3/2}, \tag{25}$$

$$\dot B_\varphi(r) = \dot B_0 r^{\frac{\lambda-4}{2}}, \tag{26}$$

where $\dot\rho_0$ and $\dot B_0$ are constant. Since we have not applied the effects of wind in the momentum and energy equations, we will assume a no wind case, $\dot\rho = 0$ and $\lambda = -3/2$. In this case, $\dot B_\varphi \propto r^{-11/4}$, which implies that creation/escape of magnetic field increases with approaching to central object. This property is qualitatively consistent with previous studies of accretion flows (Machida et al. 2006; Oda et al. 2007; Faghei & Mollatayefeh 2012).

Using the self-similar solutions in the continuity, radial momentum, angular momentum, convection parameter, energy, and induction equations [(1)-(3), (13), (14), and (18)], we can obtain the following relations:

$$\dot\rho_0 = -\left(\lambda + \frac{3}{2}\right)\alpha\rho_0 c_1 \sqrt{GM_*}, \tag{27}$$

$$-\frac{1}{2}c_1^2\alpha^2 + 1 - c_2^2 + c_3\left[\lambda - 1 + \chi(1+\lambda)\right] = 0, \tag{28}$$

$$\alpha c_1 = 3(\alpha + g\alpha_c)(\lambda+2)c_3, \tag{29}$$

$$\begin{aligned}\alpha c_1 \left[\frac{1}{\gamma-1} + \lambda\right] = \\ \frac{9}{4}\alpha f\left[(1+\frac{\alpha_c}{\alpha}g)c_2^2 + \frac{2\chi}{9P_m}c_3(1+\lambda)^2\right] \\ -\alpha_c c_3(\lambda+\frac{1}{2})\left[\frac{1}{\gamma-1}+\lambda\right]\end{aligned} \tag{30}$$

$$\alpha_c = \frac{l_M^2}{4\sqrt{2}c_3(\lambda-1)^2}\sqrt{\frac{c_3(\lambda-1)}{\gamma}[\lambda(1-\gamma)-1] - c_2^2}, \tag{31}$$

$$\dot B_0 = -\frac{\alpha\lambda}{2}GM_*\sqrt{2\pi\rho_0\chi c_3}\left[2c_1 + \frac{c_3}{P_m}(1+\lambda)\right]. \tag{32}$$

Above equations express for $\lambda = -3/2$, there is no mass loss, while for $\lambda > -3/2$ mass loss (wind) exists.

4 Results

Here, similar to Zhang & Dai (2008), we will study the presence of convection in two cases: α_c as a free parameter and α_c as a variable.

4.1 *Case 1: α_c as a free parameter*

In this case, we take the convective coefficient α_c as a free parameter to discuss the effects of convection for simplicity. Examples of such solutions are presented in Figures 1 and 2.

In Figure 1, the self-similar coefficients c_1, c_2, and c_3 are shown as functions of the parameter χ. By adding the parameter χ which indicates the role of magnetic filed on the dynamics of accretion discs, we see the coefficients of radial and rotational velocities and sound speed decrease. This properties are qualitatively consistent with results of Faghei (2011). In Figure 1, we also studied the effect of convection parameter α_c on the physical variables. The value of α_c measures the strength of convective viscosity and a larger α_c denotes a stronger turbulence due to convection. Figure 1 implies that for non-zero α_c, the radial infall velocity is lower than the standard ADAF solution and for larger α_c this reduction of radial infall velocity is more evident. It can be due to decrease of efficiency of angular momentum transport by adding the convection parameter α_c (see equation 29). The profiles of angular velocity show that it decreases with the magnitude of α_c, while the sound speed increases. These properties are in accord with results of Zhang & Dai (2008).

In Figure 2, the physical variables are shown as functions of parameter χ for several values of magnetic Prandtl number. Since inverse of magnetic Prandtl number is proportional to magnetic diffusivity, $P_m \propto \eta^{-1}$. Thus, reduce of magnetic Prandtl number denotes to increase of resistivity of the fluid. The solutions in Figure 2 imply that the accretion velocity and the sound speed both increase with the magnitude of resistivity, while the rotational velocity decreases. These properties qualitatively confirm the results of Faghei (2011).

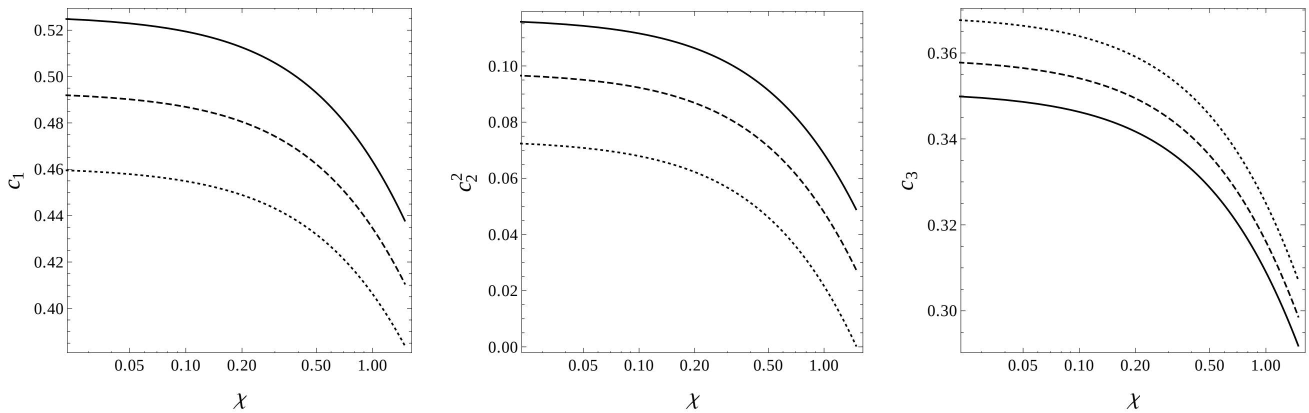

Fig. 1 Physical variables as functions of χ for several values of convective viscosity. The input parameters are set to $\alpha = 0.2$, $\gamma = 1.5$, $P_m = 1/2$, $f = 1$, $l = \sqrt{2}$, $g = -1/3$, and $\lambda = -3/2$. The solid, dashed, and dotted lines represent $\alpha_c = 0$, 0.05, and 0.1, respectively.

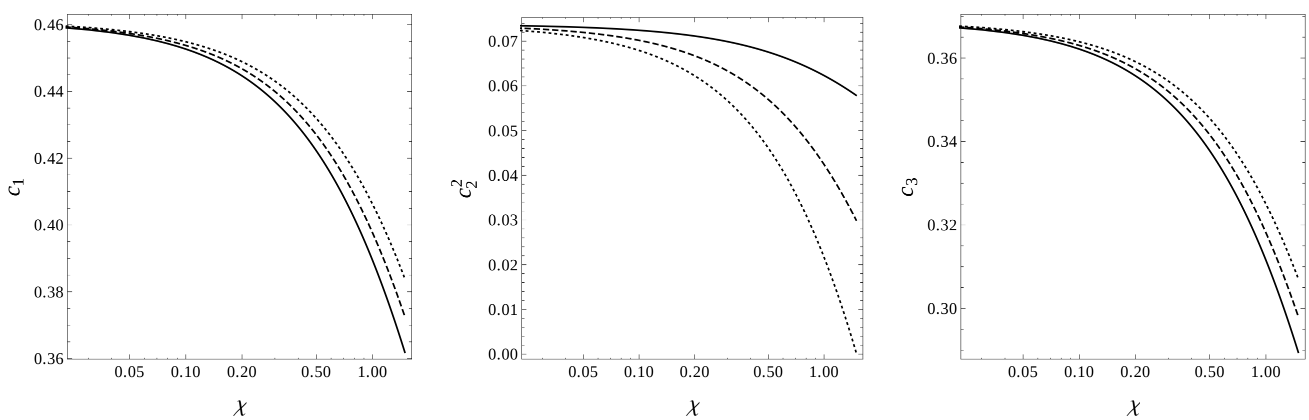

Fig. 2 Same as Figure 1, but $\alpha_c = 0.1$, and the solid, dashed, and dotted lines represent $P_m = \infty$, 1.0, and 0.5, respectively.

4.2 *Case 2: α_c as a variable*

Here, we calculate the dimensionless coefficient α_c by using the mixing-length theory. Because we used a steady self-similar method to derive α_c, it becomes a constant throughout of the accreting gas. However, it is a function of position and time (e. g. Lu et al. 2004). The amount of convection parameter α_c is calculated by equation (31). Using this equation and equation (28)-(30), we can obtain the behavior of physical quantities in the presence of convection. Such solutions are shown in Figure 3.

In Figure 3, the coefficients c_1, c_2, c_3, and convection parameter α_c are shown as functions of the degree of magnetic pressure. Similar to case 1, the accretion and rotational velocities, and sound speed decrease by adding the parameter χ. While, the convection parameter α_c increases for stronger toroidal magnetic field. This property is qualitatively consistent with result of Zhang & Dai (2008). In Figure 3, the physical variables are also studied for several values of magnetic Prandtl number. The profiles of convection parameter α_c imply that it increases by adding the magnetic diffusivity. As for non-zero magnetic diffusivity, α_c is larger than the standard CDAF solution and for larger magnetic diffusivity this increase of convection parameter α_c is more evident.

5 Summary and Discussion

The observational features of low-luminosity state of X-ray binaries and nuclei of galaxies can be successfully

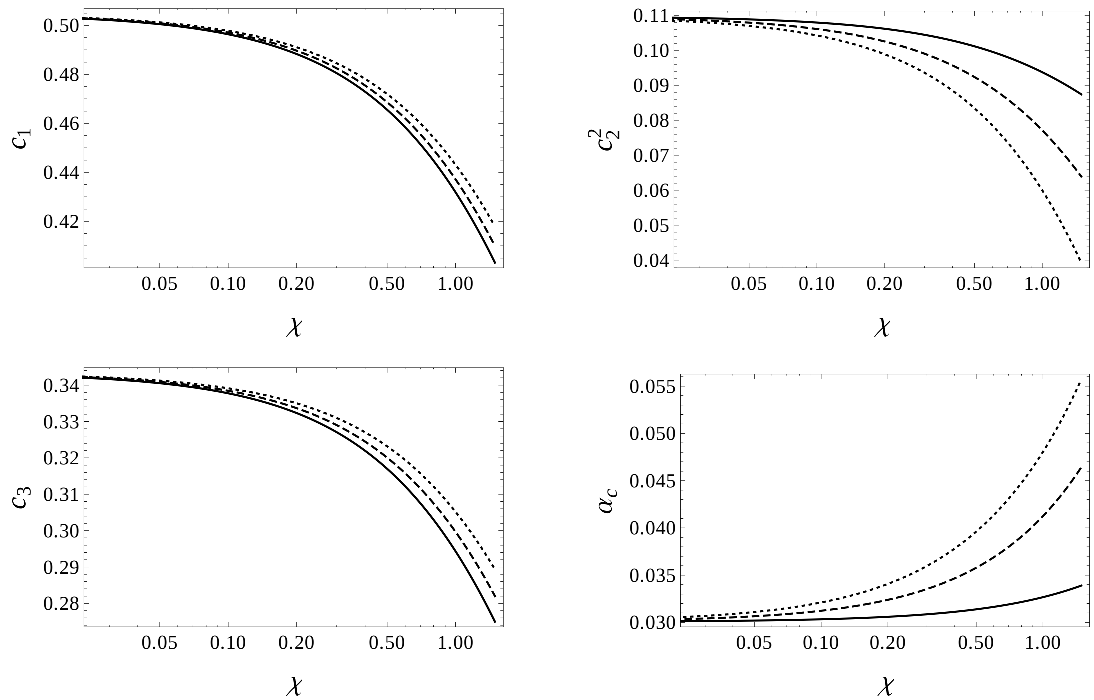

Fig. 3 Physical variables as functions of χ for several values of magnetic Prandtl number. The input parameters are set to $\alpha = 0.5$, $\gamma = 1.5$, $f = 1$, $l = \sqrt{2}$, $g = -1/3$, and $\lambda = -3/2$. The solid, dashed, and dotted lines represent $P_m = \infty$, 1.0, and 0.5, respectively.

explained by the models of radiatively inefficient accretion flow (RIAF). The importance of convection in RIAFs was realized by semi-analytical and direct numerical simulation (e. g. Narayan et al. 2000; Igumenshchev et al. 2003).

In this research, we considered the effects of convection on the presented model of Faghei (2011). Similar to Narayan et al. (2000), we assumed the convection affects on transports of angular momentum and energy. Using a radially self-similar approach, we studied the effects of convection on the model for several values of magnetic field and resistivity. The solutions showed that the accretion and rotational velocities, and sound speed decrease for stronger magnetic filed. Moreover, we found that the accretion velocity and sound speed increase with the magnitude of the resistivity, while the rotational velocity decreased. These properties are qualitatively consistent with results of Faghei (2011). We studied the effects of convection on a resistive and magnetized RIAF in two cases: assuming the convection parameter as a free parameter and using mixing length theory to calculate the convection parameter. In the first case, we found that by adding the convection parameter, the radial and rotational velocities decrease and the sound speed increases. In the second case, we found that the convection parameter increases by adding the magnetic filed and resistivity. These properties are in many aspects in accord with results of Zhang & Dai (2008).

The present model have some limitations that can be modified in the future works. For example, the latitudinal dependence of physical variables have been ignored in this paper. While, two-dimensional and three-dimensional MHD simulations of RIAFs show that the disc geometry strongly depends on magnetic field configuration (e. g. Igumenshchev et al. 2003). Thus, the study of present model in two/three dimensions can be an interesting subject for future research. Moreover, it has been understood the magnetic field can change the criterion for convective instability (e. g. Balbus & Hawley 2002). While, we igonred the effects of magnetic field on the instability criterion. Thus, the presented criterion in this paper can be modified in the future research.

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